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A Time-Inconsistency Problem in Parametric Reforms of PAYG

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Abstract: As long as there are adverse demographic shocks, the planner has to decide on the intergenerational distribution of these shocks to keep the pay-as-you-go (PAYG) system in balance. We show that any discretionary transfer policy that allocates the burden of these shocks between the elderly and the young becomes dynamically inconsistent and the system moves toward being a Ponzi scheme. Because of this time-inconsistency problem in unbinding piecewise policies, parametric reforms of PAYG tend to be unfair in terms of generational justice and could be inefficient in terms of optimal level of consumption.

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In the face of adverse demographic shocks, governments have to use parametric reforms to keep pay-as-you-go (PAYG) systems functioning, at least in the short-run. Like any other fiscal policy, the optimality of these reforms should be determined on the basis of their feasibility, generational fairness, and effects on social welfare, even if the existence of the system is not optimal. However, it is a widespread fact that these reforms around the world tend to phase in very gradually so that the senior members' rights are partially protected while more burden being shifted onto new members and coming generations. We question this bias in this study by asking whether there can be any other reason embedded in the very nature of PAYG systems other than what political equilibrium dictates for governments so that they cannot resist abusing the system by moving toward Ponzi schemes.

We show in this study that there might be a time-inconsistency problem in parametric reforms of PAYG systems so that they tend to be unfair in terms of generational justice and could be inefficient in terms of the optimal level of consumption. As long as there are adverse shocks in the system, the generational distribution of their financial burden has to be determined by the social planner. If this generational policy that allocates the burden between the elderly and the young is not binding it becomes dynamically inconsistent and the system moves toward being a Ponzi scheme.

This paper is related to a large literature on optimality of PAYG policies (Samuelson 1958, 1975; Feldstein, 1985; Marini and Scaramozzino, 1999a; Kotlikoff, 2003), on intergenerational distribution of adverse shocks (Meijdam and Verbon, 1997; Demange and Laroque, 1999), on the time-consistency of generational policies (Calvo and Obstfeld, 1988; Marini and Scaramozzino, 1999b, Willenbocked, 2008), on the political economy of intergenerational transfer institutions (Aaron 1966; Galasso 1999, 2002; Browning 1973, 1975), and on intergenerational risk sharing (Gordon, 1985; Conesa and Garriga 2007). Unlike most studies investigating distributional policies at the start-up, our work explicitly models the distributional policy implemented by a parametric reform and initiated by an adverse shock to a current PAYG system. It questions the time-consistency of this policy in a simple setting where there are dynamic interactions between the planner and rational agents as pioneered by Kydland and Prescott (1977). Section 1 describes the model by which the time- inconsistency problem in PAYG policies is discussed. Our brief interpretation of the results is presented in Section 2.

1. The Model

Since Samuelson's prominent paper appeared in 1958, it has been well-discussed that if the economy is dynamically inefficient there can be a PAYG system that improves lifetime wellbeing of every subsequent generation. As also shown by Marini and Scaramozzino (1999), even in the absence of this inefficiency, if the gain for the old, who receive benefits without paying for it, can be weighted by the society high enough to surpass the total loss for future generations, an unexpected introduction of PAYG increases the total social welfare. This occurs only if the social discount rate for future generations is high enough in the social planner's welfare function. Therefore, since an optimal social security critically depends on the level of this exogenous discount rate, the existence of PAYG can be justified entirely on ethical grounds.

When there are unexpected adverse demographic shocks in the system, the social planner has a range of policy options from the removal of the program to the full scale of Ponzi scheme. Each policy option chosen by parametric reforms has a different distribution of the burden among generations: the contribution rate can be stabilized and the whole burden of the shock can be born retirees or the benefit level can be fixed, the working population bears the entire burden.

Therefore, the determination of compromising policies between these two extreme principles should be addressed for understanding their optimality. Unlike the studies on intergenerational risk sharing that define the optimal policy as the one that minimizes the variance of the systems' internal rate of return among generations; we define the optimality of policies by their direct effects on a social welfare function without referring whether the existence of system is optimal in the first place. In using a social welfare function, this paper's efficiency criterion resembles to one used by Marini and Scaramozzino (1999) and Calvo and Obstfeld (1988). As an extension to these studies, however, we do not question the optimality of the system but look at the time-inconsistency problem in optimal policies when they are piecewise, unbinding, and evaluated based on a time-consistent social welfare function in the face of demographic shocks.

To analyze outcomes of different policy responses in PAYG systems, we use a simple two-period overlapping generation model where there is no productive capital and identical individuals are endowed by w for their inelastic labor supply. They save (s) in the first period, and retire and spend (c) their saving in the second period. To simplify further, we assume that there is no uncertainty in lifetime, liquidity constraint, bequest, and growth in endowment. To avoid defining the utility function explicitly, we initially assume that the real interest rate (r) is equal to the personal discount rate (ρ). The only function of the government is to organize a balanced PAYG system where the young, y, pay taxes, tx, and the old, o, receive benefits, b as defined below:

$$R_{t-1}b_{t-1} = L_{t-1}tx_{t-1} \Rightarrow b_{t-1} = (1+n_{t-1})tx_{t-1}$$

where R and L stand for numbers of retirees and workers respectively. The population grows by n and we introduce an adverse demographic shock (θ) to the system, at time t-1 as follows.

$$b^*_{t-1} = (1+n_{t-1}-\theta_{t-1})tx_{t-1} \Rightarrow b_{t-1} > b^*_{t-1} \tag{1}$$

We want R_t to be equal to L_{t-1} to avoid heterogeneity among retirees, that's why we assume that adverse shocks are due to negative changes in fertility.[1] Since shocks have permanent effects on populations, $n_t = n_{t-1} - \theta_{t-1}$. To see the burden explicitly and policy options for the planner to distribute it among generations, we assume that taxes are fixed and transfers are used to balance the system as follows:

$$R_{t-1}b_{t-1} = L_{t-1}tx + L_{t-1}tr_{t-1}$$

$$b_{t-1} = (1+n_{t-1}-\theta_{t-1})tx + (1+n_{t-1}-\theta_{t-1})tr_{t-1}$$

When $\theta_{t-1}tx = (1+n_{t-1}-\theta_{t-1})tr_{t-1}$, $b_{t-1} = b^*_{t-1}$. We introduce a policy variable, δ, to the model as follows.

$$tr_{t-1} = \delta_{t-1}\theta_{t-1}tx(1+n_{t-1}-\theta_{t-1})^{-1} \tag{2b}$$

[1] If we include shocks to labor force participation and longevity, the aggregation becomes more complex.

This policy variable is chosen by the planner to distribute the burden (θtx) among generations such that as it approaches to one the full burden will be shifted from the old to the young. Since it represents the planner's policy only for the system, the upper and lower limits of the transfer policy cannot exceed the removal of the system and the full scale Ponzi scheme as expressed below.[2]

$$\frac{\theta - 1}{\theta} \leq \delta < 1$$

Our setting is different from the conventional way of modeling generational transfers at the start-up of PAYG. First, when there is a shock to the system, an optimal parametric reform can be explicitly identified by optimal policy variable δ. Second, even if expectations on future shocks are zero, as long as $\delta > 0$ in the first period, every succeeding generation may have expectations on δ in the second period conditional on its value in the first period. Third, in the face cumulating shocks each generation faces higher transfers and a binding policy (a constant δ) may not be sustainable, as stated by (2c) below.

$$tr_s = \frac{tx\delta_s \sum_{i=t-1}^{s} \theta_i}{\left(1 + n_{t-1} - \sum_{i=t-1}^{s} \theta_i\right)} \tag{2d}$$

From the individual's perspective, generational transfers are recognized when they are required to pay *tr* and form their expectations in the first period about whether or not the system will be fair in the second period. However, which generation transfers, and how much, will be determined by the government's generational policy in the second period, which can be either discretionary or governed by a rule. Hence, the total consumption at time *t* will be affected by three factors: (1) the expectation of the old formed at time *t*-1, (2) the government's generational policy at time *t*, and (3) the expectation of the young at time *t*.

Formally, every young person who is required to pay taxes, *tx*, and an additional transfer, tr_{t-1} faces the following problem:

$$Max_c u(c_{yt-1}) + (1+\rho)^{-1} E_{t-1} u(c_{ot})$$

subject to

$$c_{yt-1} = w - tx - tr_{t-1} - s_{t-1}$$

and

$$E_{t-1} c_{ot} = (1+r)s_{t-1} + E_{t-1}(b_t)\,.$$

[2] We assume that the policy variable can be used only for balancing the system. If lower or higher transfers are needed to improve social welfare, another transfer policy out of the system can take care of it.

With usual assumptions in the utility function, the young at time t-1 solves this problem with the following values:

$$c_{yt-1} = \frac{1+r}{2+r}\left[w - tx - tr_{t-1} + E_{t-1}b_t(1+r)^{-1}\right] \tag{3a}$$

$$s_{t-1} = 1/(2+r)\left[w - tx - tr_{t-1} - E_{t-1}b_t\right]. \tag{3b}$$

Expectations in the first period will determine the saving. The young can expect a fair ($E_{t-1}b_t/(1+r) - tx_{t-1} - tr_{t-1} \geq 0$) or unfair PAYG ($E_{t-1}b_t/(1+r) - tx_{t-1} - tr_{t-1} < 0$). The government's generational policy will determine b_t and the consumption in the second period:

$$c_{ot} = (1+r)s_{t-1} + b_t = \frac{1+r}{2+r}\left[w - tx - tr_{t-1} - E_{t-1}b_t\right] + b_t\,. \tag{3c}$$

Moreover, the young's consumption at time t is also determined by the expectation on social security as shown below.

$$c_{yt} = \frac{1+r}{2+r}\left[w - tx - tr_t + E_t b_{t+1}(1+r)^{-1}\right]. \tag{3c}$$

Given all individuals recognize that the planner commits to a balanced-budget (2a), the young's expectation on benefits at t-1 becomes:

$$E_{t-1}b_t = (1+n_t - E_{t-1}\theta_t)tx + (1+n_t - E_{t-1}\theta_t)E_{t-1}tr_t\,. \tag{4a}$$

Using the fact that $(1+n_t) = (1+n_{t-1} - \theta_{t-1})$ and (2c), we obtain

$$(1+n_t - E_{t-1}\theta_t)E_{t-1}tr_t = E_{t-1}\delta_t(\theta_{t-1} + E_{t-1}\theta_t)tx\,. \tag{4b}$$

Hence, (4a) can be expressed in terms of taxes as follows

$$E_{t-1}b_t = tx(1+n_{t-1}) - tx(1 - E_{t-1}\delta_t)(\theta_{t-1} + E_{t-1}\theta_t)\,. \tag{4c}$$

Likewise, the transfer that the young pay at time t-1 is

$$tr_{t-1} = \frac{\delta_{t-1}\theta_{t-1}tx}{1+n_{t-1} - \theta_{t-1}}\,. \tag{4d}$$

Substituting (4c) and (4d) into (3a) and assuming $E_{t-1}\theta_t = 0$, we obtain

$$c_{yt-1} = \frac{1}{2+r}\left[(1+r)w + (n_{t-1} - \theta_{t-1} - r)tx - \frac{(1+r)\theta_{t-1}tx\delta_{t-1}}{1+n_{t-1} - \theta_{t-1}} + \theta_{t-1}txE_{t-1}\delta_t\right] \tag{4e}$$

and

$$c_{ot} = (1+r)\left[w - tx - \frac{\delta_{t-1}\theta_{t-1}tx}{1+n_{t-1}-\theta_{t-1}} - c_{yt-1} \right] + b_t \,, \tag{4f}$$

where b_t is determined by the planner as follows:

$$b_t = tx(1+n_{t-1}) - tx(1-\delta_t)(\theta_{t-1}+\theta_t)\,. \tag{4g}$$

In order to see the relationship between expectations and the government's generational policy, we assume, for a moment, that $r = n_{t-1} = 0$ and $E_{t-1}\theta_t = \theta_t = 0$.[3] By these assumptions and using (4e), (4f), and (4g) the consumption functions of current and future generations at t-1 become

$$c_{ot-1} = \frac{1}{2}w - \theta_{t-1}tx(1-\delta_{t-1}(t-1)) \tag{5a}$$

$$c_{ys} = \frac{1}{2}(w-\theta_{t-1}tx) - \frac{1}{2}\theta_{t-1}tx\left[\frac{\delta_s(s)}{1-\theta_{t-1}} - E_s\delta_{s+1} \right] \tag{5b}$$

$$c_{os+1} = \frac{1}{2}(w-\theta_{t-1}tx) - \theta_{t-1}tx\left[\frac{1}{2}\frac{\delta_s(s)}{1-\theta_{t-1}} + \frac{1}{2}E_s\delta_{s+1} - \delta_s(s-1) \right] \tag{5c}$$

To detect time inconsistency, we define the same policy determined at two subsequent times: $\delta_s(s\text{-}1)$ and $\delta_s(s)$. The optimal level of δ will be determined by the planner's objective function, which is a time-consistent[4] utilitarian welfare function for all existing and future generations as follows:

$$\begin{aligned} SW_{t-1} = {} & \frac{1+\beta}{(1+\rho)(1+n_{t-1}-\theta_{t-1})} u_o(c_{ot-1}) + \\ & \sum_{s=t-1}^{\infty} \left(\frac{1+n_{t-1}-\theta_{t-1}}{1+\beta} \right)^{s-t-1} [u_y(c_{ys}) + (1+\rho)^{-1} u_o(c_{os+1})] \end{aligned} \,, \tag{6}$$

where β, the exogenous social discount rate, captures the relative weight that the planner places between present and future generations. We assume that it is in the range that satisfies (2c) and $(n_{t-1}\text{-}\ \theta_{t-1}) < \beta$. By our assumptions, since there is no substitution between leisure and consumption, transfers are nondistortionary. In addition, we do not have productive capital so

[3] These assumptions mean that $r > n_t$ and the population growth is negative. Therefore, without a significant discount rate, the existence of the system reduces the wellbeing of all generations and always there is a tradeoff between generations welfare.

[4] The reverse discounting for the old is a necessary condition for a symmetric treatment between generations as explained by Calvo and Obstfeld (1988).

that a reduction in saving has no effects on factor prices; hence, the change in the consumption for each generation is the only consequence.

Given that that $r = n_{t-1} = 0$ and $E_{t-1}\theta_t = \theta_t = 0$, if the distribution policy is binding, (5a), (5b), and (5c) become

$$c_{ot-1} = \frac{1}{2}w - \theta_{t-1}tx(1-\delta) \tag{7a}$$

$$c_{ys} = c_{os+1} = \frac{1}{2}(w - \theta_{t-1}tx) - \frac{1}{2}\frac{\delta\theta_{t-1}^2 tx}{1-\theta_{t-1}} \tag{7b}$$

It is obvious from this setting that any policy with $\delta > 0$ can be feasible and that distributions from unborn to current generations can be increased as β goes up.[5] As seen in (7b), when δ is zero, since the number of young person is (1-θ) times as less as the old, keeping the system functioning will cost next generations by $\theta_{t-1}tx$. As δ gets higher than zero, this cost will increase by the second term in (7b) for the same reason. The planner's problem is to find the optimal distribution of the burden that maximizes social welfare. To see the explicit solution to this problem, we assume that the utility index is $u(c) = ln(c)$. Since ρ and n_{t-1} are zero and using the fact that $c_{ys-1} = c_{os}$, (6) can be reduced to

$$\begin{aligned} SW_{t-1} &= (1+\beta)\ln\left[\frac{1}{2}w - \theta_{t-1}tx(1-\delta)\right] \\ &+ (1-\theta)2\ln\left(\frac{1+\beta}{\beta-\theta_{t-1}}\right)\frac{1}{2}\left[(w-\theta_{t-1}tx) - \theta_{t-1}tx\frac{\delta\theta_{t-1}}{1-\theta_{t-1}}\right] \end{aligned} \tag{8}$$

According to (8), the marginal effect of δ on social welfare can be expressed as follows

$$\begin{aligned} \frac{\partial}{\partial\delta}SW_{t-1} &= (1+\beta)\theta tx\left[\frac{1}{2}w - \theta tx(1-\delta)\right]^{-1} \\ &- \frac{1}{2}\left(\frac{1+\beta}{\beta-\theta}\right)\theta tx\left[(w-\theta tx) - \theta tx\frac{\delta\theta}{1-\theta}\right]^{-1} \end{aligned} \tag{9}$$

where we ignore the subscript of the shock. Using (9), the optimal δ can be found as

$$\delta = \frac{(1-\theta)\left[w(\beta-2\theta) - \theta tx(\beta-3\theta)\right]}{\theta^2 tx\left[-3\theta+\beta\right]}$$

It is clear from (10) that δ can be positive only if

[5] Even if social discount rate is zero and equal to personal discount rate, it can be argued that setting δ to zero or less than zero (removal of the program) becomes unfair for the current old. This can be seen by (5a), (5b), and (5c) that when $\delta_{t-1} = 0$, even if it seems that each generation loses the same amount of consumption, $\theta_{t-1}tx$, the lost utility for the old should be more than others because of their inability to smooth this loss backward.

$$\beta > \frac{2\theta w - 3\theta^2 tx}{w - \theta tx} \tag{11a}$$

When the social discount rate is lower than this threshold, δ turns out to be negative and the planner can question the removal of the program by setting δ to $(\theta\text{-}1)/\theta$.

When the policy is not binding, it is said to be time-consistent only if the same policy set at time t is still optimal for the government at time $t+s$ after the expectations are formed, as expressed below:

$$\delta_{t+s}(t+s) = \delta_{t+s}(t) \tag{12}$$

To check whether this condition holds, we question the existence of any incentive for the planner to renege its previously set policy. When there is a shock in the system at time t-1, first, the planner decides on $\delta_{t\text{-}1}$ that satisfies (5). Then, having faced $tr_{t\text{-}1}$, the young set their expectations on δ_t at time t-1. After expectations are formed the government chooses $\delta_t(t)$, which determines tr_t. Following these steps, we reset the consumption function for the current old at time t below.

$$c_{ot} = \frac{1}{2}(w - \theta_{t-1}tx) - \theta_{t-1}tx\left[\frac{1}{2}\frac{\delta_{t-1}(t-1)}{1-\theta_{t-1}} + \frac{1}{2}E_{t-1}\delta_t - \delta_t(t)\right]$$

The expectations are set as $E_{t\text{-}1}\delta_t = \delta_{t\text{-}1}(t\text{-}1)$ at time t-1. Since the planner's announced policy is always $\delta_s(s) = \delta_{s+1}(s)$, we assume that $E_t\delta_{t+1} = \delta_t(t)$. Hence we can rewrite the consumption functions (12), (5b) and (5c) as follows:

$$c_{ot} = \frac{1}{2}(w - \theta_{t-1}tx) - \theta_{t-1}tx\frac{1}{2}\delta_{t-1}(t-1)\left(\frac{2-\theta_{t-1}}{1-\theta_{t-1}}\right) + \theta_{t-1}tx\delta_t(t)$$

$$c_{yt} = c_{ot+1} = \frac{1}{2}(w - \theta_{t-1}tx) - \frac{1}{2}\frac{\delta_t(t)\theta_{t-1}^2}{1-\theta_{t-1}}$$

Since expectations for unborn generations cannot be expressed, their consumption functions will be reduced to (13c). Using the same welfare function, (6), with the same social discount rate, $\delta_t(t)$ can be obtained as below:

$$\delta_t(t) = \frac{(1-\theta)\left[w(\beta - 2\theta) - \theta tx(\beta - 2\theta)\right]}{\theta^2 tx\left[2 - 3\theta + \beta\right]} + \frac{(2-\theta)(1-\theta)}{2-3\theta+\beta}\delta_{t-1}(t-1)$$

As long as (11a) holds, $\delta_t(t) > \delta_t(t\text{-}1)$. However, when the social discount rate is equal to the threshold given by (11a), $\delta_{t\text{-}1}(t\text{-}1)$ ($= \delta_t(t\text{-}1)$) is set to zero and $\delta_t(t)$ becomes negative.[6] This formal presentation of time-inconsistency needs intuitive explanation. The time-inconsistency in

[6] Unless (11a) holds, $\delta_t(t) < \delta_t(t\text{-}1)$

optimal policies of PAYG arises from the fact that the consumption in the second period depends on the saving decision made in the first period. Therefore, the consumption at time *t* is a negative function of benefits expected at time *t*-1 as seen by (3c). If the social discount rate is large enough to justify a positive policy variable, once the saving decision is made in the first period, since it is not expected, any increase in the benefit has a positive effect on the consumption in the second period. If the discount rate is insignificant for positive policy variable, transfers from the young to the old become negative. Again, once the saving decision is made in the first period, since it is not expected, the planner can improve the welfare by increasing the negative transfers.

To see the time-inconsistency problem on the aggregate level of consumption, we assume, for a moment, that

$$r = n_t - \theta_t = 0,$$

and

$$E_{t-1}\theta_t = \theta_t = 0.$$

With these assumptions (4e) becomes:

$$c_{yt-1} = \frac{1}{2}\left[v - \theta_{t-1}tx\delta_{t-1} + \theta_{t-1}txE_{t-1}\delta_t\right]. \tag{8a}$$

Since $c_{ot} = (1+r)s_{t-1} + b_t$ we obtain

$$c_{ot} = (1+r)\left[v - tx + \theta_{t-1}tx\delta_{t-1} - c_{yt-1}\right] + b_t, \tag{8b}$$

where $b_t = tx - \theta_{t-1}tx\delta_t$. Similar to (4e), the young also have the following consumption function at time *t*+1:

$$c_{yt} = \frac{1}{2}\left[v - \theta_{t-1}tx\delta_{t-1} + \theta_{t-1}txE_t\delta_{t+1}\right]. \tag{8c}$$

To aggregate (8b) and (8c), we use

$$L_t = (1 + n_{t-1} - \theta_{t-1})^2 L_{t-2}.$$

If we normalize L_{t-2} (i.e. R_{t-1}) to one before and, by assumptions, $\theta_t = 0$ and $n_t = n_{t-1} - \theta_{t-1} = 0$, it becomes

$$L_t = (1 + n_{t-1} - \theta_{t-1})^2 L_{t-2} = R_t = 1$$

By substituting (8b) and (8c) into

$$C_t = R_t c_{ot} + L_t c_{yt}$$

we obtain the following total consumption at time t:

$$C_t = w + \frac{1}{2}\theta_{t-1} tx(E_t\delta_{t+1} - E_{t-1}\delta_t) + \frac{1}{2}\theta_{t-1} tx(\delta_t - \delta_{t-1}) . \tag{8d}$$

If the policy is binding and sustainable, that is,

$$E_{t-1}\delta_t = E_t\delta_{t+1} = \delta_{t-1} = \delta_t ,$$

it becomes irrelevant, as expected, in terms of its effect on consumption in the second and subsequent periods. However, if the policy is not binding, after expectations are set as $E_{t-1}\delta_t = \delta_{t-1}$ any policy, which is $E_{t-1}\delta_t = \delta_{t-1} < \delta_t$, increases total consumption and, therefore, it results in time-inconsistency as $\delta_t(t) \neq \delta_t(t-1)$ in optimal policies of PAYG.

The time-inconsistency problem can be stated formally as follows. After normalizing R_t to 1, the government wants to maximize the following social welfare function at time t:

$$SW_t = (1+\beta)R_t u_o(c_{ot}) + L_t u_y(c_{yt}) + R_{t+1} u_o(c_{ot+1}) + A_t \tag{8a}$$

subject to (2c), (2d), and

$$c_{ys-1} = \frac{1}{2+r}\left[(1+r)w + (n - \theta_{t-1} - r)tx - \frac{(1+r)\theta_{t-1} tx\delta_{s-1}}{1+n_{t-1}-\theta_{t-1}} + \theta_{t-1} txE_{s-1}\delta_s\right],$$

$$c_{os} = (1+r)\left[w - tx - \frac{\delta_{s-1}\theta_{t-1} tx}{1+n_{t-1}-\theta_{t-1}} - c_{ys-1}\right] + b_s .$$

The last part of (8a), the sum of utilities for future generations after t+1, A, represents the present value of utility indexes for all future generations calculated by the social discount rate.[7] If the reverse discounting for the old at time t is omitted, the argument becomes

$$SW_t = \lambda_t\{u_{yt}(E_t\delta_{t+1}, \delta_t) + u_{ot+1}(E_t\delta_{t+1}, \delta_t, \delta_{t+1})\} + u_{ot}(E_{t-1}\delta_t, \delta_{t-1}, \delta_t) + A_t$$

The planner chooses its policy variable, δ_t, that satisfies the following first-order condition:

$$\lambda_t(u'_{yt} + u'_{ot+1})\left[\frac{dE_t\delta_{t+1}}{d\delta_t} + 1\right] + u'_{ot}\left[\frac{dE_{t-1}\delta_t}{\partial\delta_t} + 1\right] + \frac{dA_t}{d\delta_t} = 0$$

[7] This is because present policies cannot affect expectations of unborn generations at time t.

Following the argument pioneered by Kydland and Prescott (1977), in order for an optimal policy to be consistent, the effect of δ_t on $E_{t-1}\delta_t$ should be zero,[8] which is not possible if the policy is not committed to a rule. The reason for this failure is the existence of the incentive for the planner to renege its generational policy after the young form their expectations. As long as the young have the knowledge of the fact that the planner has an incentive to apply $\delta_t(t)$ higher than $\delta_t(t\text{-}1)$, they set their expectations as to receive a fairer PAYG. Once the expectations at time t-1 are set to receive a fairer PAYG, any discretionary policy which reduces consumption by setting an unfair (or less fair) PAYG may not be feasible. As a result, the entire system moves toward a full-scale Ponzi scheme.

2. Conclusion

The time-inconsistency problem could partly explain how the generosity of the system and the decreasing expectations in social security wealth exist together. It may also help us understand how, as people get older, they become proponents of the system, even if their life-time social security wealth is negative. A decreasing trend in the expectations from PAYG creates an irresistible incentive for governments to abuse the system by reneging the policy set before. Therefore, as long as it is discretionary and not a rule, even in our partial equilibrium setting, a strictly neutral generational policy turns out to be unsustainable. In other words, the government employs populist policies to increase the consumption by giving 'some' to the old and promising 'some' to the young at time t. In this case, the young members may still expect unfair PAYG to the extent in which government tries its best to be 'fair'. As the government approaches to a hundred-percent fair PAYG and convince people that it would honor its promises, the system grows to be a full-scale Ponzi scheme. As a result, an attempt to make PAYG fairer by implementing a new policy, which distributes the burden of adverse shocks among generations by setting δ_t lower than one, ends up with a higher level of consumption.

[8] Or the marginal utility of the old (u'_{ot}) should be zero.

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